

MATH3705 Tutorial 7

Determine the following:

1. $\mathcal{F}(e^{-0.5|x|})$.

Solution: Let $f(x) = e^{-|x|}$. Then $\mathcal{F}(e^{-0.5|x|}) = \mathcal{F}(f(0.5x)) = \frac{1}{0.5} \hat{f}\left(\frac{\lambda}{0.5}\right) = \frac{4}{1+4\lambda^2}$.

2. $\mathcal{F}(e^{-|x-4|})$.

Solution: Let $f(x) = e^{-|x|}$. Then $\mathcal{F}(e^{-|x-4|}) = \mathcal{F}(f(x-4)) = e^{i4\lambda} \hat{f}(\lambda) = \frac{2e^{i4\lambda}}{1+\lambda^2}$.

3. $\mathcal{F}(e^{-ix}e^{-|x|})$.

Solution: Let $f(x) = e^{-|x|}$. Then $\mathcal{F}(e^{-ix}e^{-|x|}) = \hat{f}(\lambda-1) = \frac{2}{1+(\lambda-1)^2}$.

4. $\mathcal{F}(e^{2ix-|x+3|})$.

Solution: $\mathcal{F}(e^{-|x+3|}) = \frac{2e^{-i3\lambda}}{1+\lambda^2}$, $\Rightarrow \mathcal{F}(e^{2ix-|x+3|}) = \mathcal{F}(e^{2ix}e^{-|x+3|}) = \frac{2e^{-i3(\lambda+2)}}{1+(\lambda+2)^2}$.

5. $\mathcal{F}^{-1}\left\{\frac{e^{-i3\lambda}}{1+\lambda^2}\right\}$.

Solution: $\mathcal{F}^{-1}\left\{\frac{e^{-i3\lambda}}{1+\lambda^2}\right\} = \frac{1}{2}e^{-|x+3|}$.

6. $\mathcal{F}^{-1}\left\{\frac{1}{1+(\lambda+5)^2}\right\}$.

Solution: $\mathcal{F}^{-1}\left\{\frac{1}{1+(\lambda+5)^2}\right\} = \frac{1}{2}e^{5ix-|x|}$.

7. $\mathcal{F}^{-1}\left\{\frac{e^{i3\lambda}}{1+(\lambda-2)^2}\right\}$.

Solution: $\mathcal{F}^{-1}\left\{\frac{e^{i3\lambda}}{1+(\lambda-2)^2}\right\} = \frac{1}{2}e^{-2i(x-3)-|x-3|}$.